

4A *Time: 3 minutes*

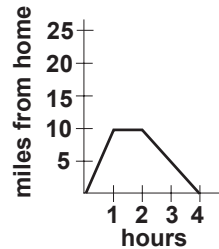
Find the sum of the integers from -2007 through $+2009$, inclusive.

4B *Time: 5 minutes*

The sum of the lengths of three sides of a rectangle is 55 cm. Each side is a whole number of cm in length. The length is 8 cm more than the width. Find the perimeter of the rectangle, in cm.

4C *Time: 4 minutes*

Jen travels from home on a straight road. After a rest she returns along the same route. The graph shows her distance from home at any given time. What was her average speed for the entire 4-hour trip, in miles per hour?



4D *Time: 6 minutes*

The pages of a book are consecutively numbered from 1 through 384. How many times does the digit 8 appear in this numbering?

4E *Time: 6 minutes*

The product of 180 and the positive integer N is a perfect cubic number. What is the least possible value of N ?

Please fold over on line. Write answers on back.



SOLUTIONS AND ANSWERS

4 A

Items in parentheses are not required.

- 4A** *Strategy:* Use the property of additive inverses (opposites). Write the sum as $(-2007 + -2006 + \dots + +2006 + +2007) + +2008 + +2009$. The sum in parentheses is 0, and the sum of all the integers from -2007 through 2009 is $+2008 + +2009 = 4017$.

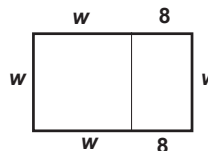
4017

- 4B** *Strategy:* Draw a picture and examine the possibilities.

There are two possibilities. Either \square or \square is 55 cm.

The first case is not possible. If the sum of 8 and three widths is 55 cm, then the width is not an integer.

The second case is possible. If the sum of 16 and three widths is 55 cm, then the width is 13 cm. This makes the length 21 cm, and the perimeter of the rectangle 68 cm.



4 B

68
(cm)

FOLLOW-UPS: (1) Find a value to replace the 55 so that the first case results in an integer value but the second does not. [Any of the numbers 11, 14, 17, and so on.] (2) Can you find a value that satisfies both situations? [No]

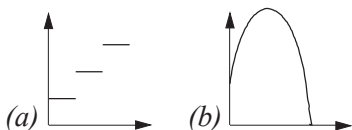
4 C

- 4C** *Strategy:* Use the definition of average speed.

Average speed is the total distance traveled divided by the total time elapsed. The graph shows that Jen traveled 10 miles from home before she rested and then traveled 10 miles back. The total time was 4 hours, so her average speed was $(10 + 10) \div 4 = 5$ mph.

5
(mph)

FOLLOW-UP: Write a situation that could be represented by each of these graphs:



[Many situations are possible. For example: The step graph, *a*, might represent the number of buses needed for a large group of people. The parabola, *b*, might represent the height over time of a ball tossed upwards.]

4 D

73
(times)

- 4D** *Strategy:* Count in an organized way.

METHOD 1: *Strategy:* Examine all the ones and tens digits separately.

There is 1 ones digit of 8 in every set of 10 consecutive numbers. From 1 through 384, there are 38 complete sets of 10 and therefore the digit 8 appears 38 times as a ones digit.

There are 10 tens digits of 8 in every set of 100 consecutive numbers. From 1 through 384, there are 3 complete sets of 100 (1-100, 101-199, 200-299) containing 30 tens digits of 8. In addition, the numbers 380-384 contain 5 more tens digits of 8. In all, the digit 8 appears $38 + 30 + 5 = 73$ times.

4 E

(N =)
150

Olympiad 4 , Continued

METHOD 2: *Strategy:* Count by intervals, treating the 80s as a special case.

Interval	1-79	80-89	90-179	180-189	190-279	280-289	290-379	380-384
# of 8s	8	11	9	11	9	11	9	5

The digit 8 appears $8 + 11 + 9 + 11 + 9 + 11 + 9 + 5 = 73$ times.

4E *Strategy:* Examine the prime factors.

$180 = 2 \times 2 \times 3 \times 3 \times 5$. Suppose $180 \times N = R \times R \times R$. Any prime factor of the product is a factor of R and thus occurs 3 times, once in each R . That is, each prime factor of a perfect cube occurs 3 (or 6 or 9 or ...) times. Then the least perfect cube, $180 \times N$, needs the additional factors 2, 3, 5, and 5. **The least possible value of N is $2 \times 3 \times 5 \times 5 = 150$.**

FOLLOW-UPS: (1) What is the least N greater than 150 for which the product of 180 and N is a perfect cube? [1200] (2) What is the least positive integer N for which the product of 12 and N is a perfect fourth power? [108] (3) What are the three least integers which are both perfect squares and perfect cubes? [0, 1, 64; they are actually sixth powers.]

NOTE: Other FOLLOW-UP problems related to some of the above can be found in our books "Math Olympiad Contest Problems for Elementary and Middle Schools" and "Creative Problem Solving in School Mathematics."