

**5A** Time: 4 minutes

Ten posts are placed 6 meters apart in a straight line. A fence runs from the first post to the last post. How long is the fence?

(Ignore the thickness of the posts.)

5B Time: 5 minutes

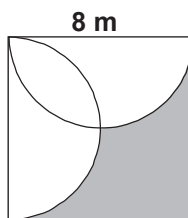
The product of three consecutive whole numbers is 15,600. What is the smallest of the three numbers?

5C Time: 6 minutes

72 more females than males are enrolled in Archimedes High School. 60% of the students are female. What is the number of male students that are enrolled?

5D Time: 7 minutes

Two semicircles are inscribed in a square with side 8 m, as shown. Approximate the area of the shaded region *to the nearest tenth of a sq m*. Use the approximation 3.14 for π .

**5E** Time: 5 minutes

The value of a two-digit number is 10 more than 3 times the sum of its digits. The units digit is 1 more than twice the tens digit. Find the two-digit number.

Please fold over on line. Write answers on back.

Division

M

Mathematical Olympiads



MARCH 8, 2006

for Elementary and Middle Schools

Contest

5

5A

Student Name and Answer

meters

5B

Student Name and Answer

5C

Student Name and Answer

male students

5D

Student Name and Answer

sq m

5E

Student Name and Answer

Please fold over on line. Write answers in these boxes.



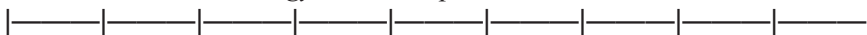
SOLUTIONS AND ANSWERS

5A

Items in parentheses are not required.

- 5A METHOD 1:** *Strategy:* Start with a simpler case and look for a pattern. 2 posts result in a fence 1×6 or 6 m long; 3 posts result in a fence 2×6 or 12 m long. Then **10 posts result in a fence 9×6 or 54 m long.**

METHOD 2: *Strategy:* Draw a picture.



There are ten posts and nine pieces of fence between them. The fence is 6×9 or 54 m long.

FOLLOW-UP: How many fence posts are required to enclose a rectangular garden that is 12 m by 16 m if the posts are placed 4 m. apart? [14]

54

(meters)

5B

24

- 5B METHOD 1:** *Strategy:* Examine the prime factors.

The prime factors of 15,600 are $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 13$. Of three consecutive numbers, no more than one can be a multiple of 5. Since there are two 5s in the list of prime factors, they must come from the same number, a multiple of 25. One of the numbers is a multiple of 13. This suggests rearranging the prime factors as $(2 \times 13) \times (5 \times 5) \times (2 \times 2 \times 2 \times 3)$. Since $26 \times 25 \times 24 = 15,600$, **the smallest of the three numbers is 24.**

METHOD 2: *Strategy:* Use the fact that the three factors are nearly equal. Since $10^3 = 1000$, $20^3 = 8000$, and $30^3 = 27,000$, the numbers are between 20 and 30. One of the numbers must be a multiple of 5, so try 25^3 , which is 15,625. Therefore, one possible answer could be $24 \times 25 \times 26$. Since it gives us the correct product of 15,600, the smallest of the three numbers is 24.

5C

144

(male students)

- 5C METHOD 1:** *Strategy:* Find the percent represented by the given amount.

Since 60% of the students are female, then 40% are male. The number of females exceeds the number of males by $(60 - 40)$ or 20% of the total number of students. Then 72 is 20%, or one-fifth, of the total. The total number of students is 5×72 , or 360. Thus, 40% of 360, or **144 male students, are enrolled.**

METHOD 2: *Strategy:* Try simpler related cases and look for a pattern. Assuming a total of 100, 200, and 300 students, this table shows in each case the subtraction of the number of males from the number of females.

# of students	# of females (60%)	# of males (40%)	Difference
100	60	40	20
200	120	80	40
300	180	120	60

Notice that in each case the number of males (column 3) is twice the difference (column 4). Since the difference is actually 72, the number of male students is 144.

Method 3 on back.

5D

22.9

(sq m)

5E

49

METHOD 3: *Strategy: Use algebra.*

Let x = the number of males and $x + 72$ = the number of females.

Set up a proportion: $\frac{x+72}{x+x+72} = \frac{60}{100}$

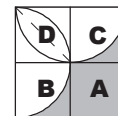
Solving for x : 144 male students are enrolled.

5D *Strategy: Split the region into more familiar shapes.*

Draw two lines as shown to divide the square into four congruent smaller squares.

METHOD 1: *Strategy: Find the area of each shaded region separately.*

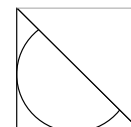
Square **A**, fully shaded, has an area of 16 sq m. In squares **B** and **C**, the shaded region is found by removing the quarter circle from the square. Each of those areas is then $16 - (\frac{1}{4} \times 3.14 \times 4^2)$, which then simplifies to 3.44 sq m. Adding 16, 3.44, and 3.44 and rounding off, **the area of the shaded region is then 22.9 sq m.**



METHOD 2: *Strategy: Find the area of the unshaded region.*

METHOD 2A: The unshaded area is the sum of the areas of regions **D**, **C**, and **B**. The area of square **D** is 16 sq m. The area of one quarter-circle is $\frac{1}{4} \times 3.14 \times 4^2$, or 12.56, sq m. The area of the unshaded region is $16 + 2 \times 12.56$, or 41.12 sq m. The area of the shaded region is then $64 - 41.12$, or 22.88, which rounds to 22.9 sq m.

METHOD 2B: Think of the unshaded region as the sum of two semicircles of radius 4 m minus the “overlap”. The overlap, divided into 2 parts by the dotted line in the diagram, consists of two regions congruent to the region considered in problem 4D. The area of that region was found as 4.56, so the area of the unshaded region is 2×4.56 less than $(2 \times \frac{1}{2} \times 3.14 \times 4^2)$. As above, the area of the shaded region is $64 - 41.12$, or about 22.9 sq m.



FOLLOW-UP: A semicircle with area 18π is inscribed in an isosceles right triangle as shown. Find the area of this triangle.[72]

5E **METHOD 1:** *Strategy: List all possible values for the tens digit.*

Consider the second condition first. The possible two-digit numbers are 13, 25, 37, and 49. The table compares the results of the first condition against those of the second condition.

Tens digit	1	2	3	4
Units digit	3	5	7	9
Number	13	25	37	49
Sum of digits $\times 3$	12	21	30	39
Difference = 10?	No	No	No	YES

The two-digit number is 49.

METHOD 2: *Strategy: Use Algebra.*

Let u = the units digit and t = the tens digit.

The equations are $10t + u = 3(t + u) + 10$ and $u = 2t + 1$.

Substituting $2t + 1$ for u in both places, we get $10t + 2t + 1 = 3(t + 2t + 1) + 10$.

Solving the equation yields $t = 4$. Then $u = 9$, and the two-digit number is 49.

NOTE: Other problems related to some of the above can be found in our books “Math Olympiad Contest Problems for Elementary and Middle Schools” and “Creative Problem Solving in School Mathematics.”